A Synopsis On

"A Study of Some Traveling Wave Solutions of Partial Differential Equations"

Submitted to

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For the Degree of

Doctor of Philosophy (Ph. D.)

In

Mathematics

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March-2022

Title: A Study of Some Traveling Wave Solutions of Partial Differential Equations

1. Introduction:

Various physical phenomena in nature are modeled by partial differential equations (PDEs) such as fluid dynamics, cosmology, finance, elastic rods, anharmonic lattices, DNA excitation, heat transfer problem, electrostatics, and other branches of science. Wave equations are a special class of partial differential equations that have been intensively studied by many researchers over the past few decades. Some phenomena where wave propagation plays a fundamental role are impulses in nerve fibers, epizootic waves in biology, combustion and detonation waves in chemistry, water waves, sound waves in air and liquid, stress waves, shock waves, earthquakes, and electromagnetic waves, etc.

One of the most important physical phenomena of the partial differential equation is traveling waves. Traveling waves have been used to demonstrate several problems in shallow-water waves, the flow of heat, optical fibers, biology, chemical kinematics, plasma physics, electricity and quantum mechanics, etc. Therefore, constructing traveling wave solutions of partial differential equations play a significant role in the development and proper understanding of qualitative features of many phenomena and models in various areas of science. Traveling wave solutions of PDEs gain much more attention from many researchers and have a wide scope of research. The study of traveling wave solutions includes the soliton solutions, periodic solutions, peakons solutions, compacton solutions, cnoidal solutions, etc. A solitary wave is one of the special types of traveling wave which was introduced by John Russell [1]. Solitons are nonlinear solitary waves caused by a cancellation of nonlinear and dispersive effects in the medium. Solitary waves maintain their shape and size while traveling with constant velocity. Also, when two solitary waves interact with each other they retain their shapes. It has been observed that solitary waves play a vital role in the study of different phenomena like surface gravity waves, nonlinear acoustic of bubbly liquids, nonlinear optics, magma flow, biophysics, plasmas, field theory, water waves, Bose-Einstein condensates, etc.

2. Review of literature and development in the subject:

Many methods exist in the literature for finding approximate and exact solutions of partial differential and integral equations viz Tanh method [2], LDM [3], Sine-Cosine method [4], Inverse scattering method [5], Hirota bilinear

method [6], Exp-Function method [7], Bäcklund transformation [8], ADM [9] and so on.

M. Younis and co-authors used the solitary wave ansatz method to find bright soliton, dark soliton, and singular soliton solutions of a nonlinear model of nanobioelectronics transmission lines [10]. H. Baskonus and co-authors found various types of traveling wave solutions of the (2+1) dimensional nonlinear electrical transmission line model that are useful in telecommunications lines [11]. A. M. Wazwaz obtained soliton solutions for various forms of the fifth-order nonlinear KdV equation which include the Lax, Sawada–Kotera (SK), Sawada–Kotera–Parker–Dye (SKPD), Kaup–Kupershmidt (KK), Kaup–Kupershmidt–Parker–Dye (KKPD), and the Ito equations by using extended tanh method [12]. R. Asokan and D. Vinodh obtained exact soliton, compacton, periodic, and traveling wave solutions of Sawada - Kotera Equation by using the tanh-coth method [13]. Bright and dark soliton solutions of the Schäfer–Wayne short-pulse equation were obtained by O. Gaxiola by the Laplace substitution method [14]. A. Seadawya and K. Rashidy constructed exact traveling wave solutions for coupled KdV equations, coupled system of variant Boussinesq equations, coupled Burger's equations, and generalized coupled KdV equations with the help of the extended algebraic method [15].

3. Objectives of Research:

The main objective of the proposed research work is:

- To study particular types of partial differential equations that exhibit traveling wave solutions.
- To find solitary wave solutions of PDEs.
- To find periodic wave solutions of various differential equations.
- To prove the existence of solutions of PDEs by some methods in the literature.

4. Methodology:

- Collection and study of some books and research papers related to the research topic.
- Analysis of research papers and their literature reviews.
- Study methods such as LDM, LSM, ADM, and some other methods.

- With the help of known methods try to solve some differential equations.
- Learn softwares like MATHEMATICA, MAXIMA, and PYTHON.
- Learn LaTeX for scientific typing.
- Visit various institutes and interact with eminent mathematicians related to our research area.
- Attend and present research articles at national, and international conferences related to our research work.

5. Importance of the study:

Nowadays finding traveling wave solutions of partial differential equations has become of prime importance in the field of applied sciences. Obtaining exact or approximate traveling wave solutions to partial differential equations is still a significant problem and has wide research scope. However, finding solutions to these equations is a difficult task. So, finding such solutions to PDEs has great significance in modern science and engineering. The present work will be useful to mathematicians, engineers, researchers, and scientists at the national and international levels. And then they can use this work to analyze and give the physical significance of various problems in the applied sciences.

6. Plan of Research work:

Sr. No.	Period	Plan of the work	Target to be achieved
1	First-year	Study of the background required for the research topic.	Good command on the subject and clear the course work.
2	Second-year	To Collect and analyze various research articles on the concerned subject. Read official documentation of required softwares.	To implement constructive techniques for various problems. Get hands-on practice with various softwares.
3	Third-year	Work on some problems to be included in the thesis.	Submit at least two research papers in UGC-CARE list journals.
4	Fourth-year	Publishing some more papers and submitting the Thesis.	Publish some more papers and Thesis submissions.

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